**My name is Simon Njeru Mugoh.**

**Welcome to CEMASTEA Mathematics Tutorial Session, in this Session we will explain ‘Area Approximation using Trapezium Rule’.**

Most shapes in real life situation are mainly irregular.

What are some of these natural shapes that are irregular?

How do you approximate the area of such irregular shapes? There are many methods and one of the methods we are going to study is **Trapezium Rule** to approximate the area of such shapes.

Let us recall the following: **What is a trapezium?**

A trapezium is a quadrilateral with one pair of parallel sides.

As you can see from the diagram b1 and b2 are 2 parallel sides and a perpendicular distance between b1 and b2 called the height, designated by letter h.

**Can we now recall how to find the area of a trapezium?**

Area of trapezium is equal to half h into b1 + b2 where h is the perpendicular distance between the two bases b1 & b2.

**Area under a curve**

Let us estimate the area of the curve as shown in the diagram below.

What is the area under the curve? The area under curve is defined by the curve itself, x-axis and two vertical lines. The vertical lines are simply the perpendicular lines issued from the two end points of the curve to the x-axes. To estimate the area under the curve by help of trapezium rule, 1st we need to divide the region into several trapezia with uniform width or height

In case we have our area divided into 4 trapezia A1, A2, A3 and A4 as indicated in the diagram, we calculate the area of each trapezium then add all of them to find an approximation of area under the curve.

From the diagram we have the length of parallel sides of each trapezium and the uniform width or height which is equal to 2. We refer to these parallel sides as **ordinates**. From the diagram we can see that the 1st ordinate is 4 units, 2nd ordinate 5 units, 3rd ordinate 6 units, 4th ordinate 4 units, and 5th ordinate 2 units.

Therefore the area under the curve is A1 + A2 + A3 + A4 where A1 is the area of the 1st trapezium or 1st strip, A2 is the area of 2nd trapezium or 2nd strip, A3 is area of the 3rd trapezium or 3rd strip and A4 is area of last trapezium or strip.

Now try to estimate the area under the given curve. Hint, use area of trapezium to calculate the area of each strip

Take 3 min to estimate the area under this curve. You are expected to pause the video for the 3 min.

**Solution**

A1 = half times 2 into 4 +5 where 2 is height (h) or width and 4 and 5 are the lengths of b1 and b2 of the 1st trapezium or strip. Thus, A1 = 9 units squared.

A2 = half times 2 into 5 + 6 where 2 is the height or width and 5 and 6 are the lengths of b1 and b2 of the 2nd trapezium or strip. Thus, A2 = 11 units squared.

A3 = half times 2 into 6+4 where 2 is the height or width and 6 and 4 are the lengths of b1 and b2 of the 3rd trapezium or strip. Thus, A3 = 10 units squared.

A4 = half times 2 into 4+2 where 2 is the height and 4 and 2 are the lengths of b1 and b2 of the last (4th) trapezium or strip. Thus, A4 = 6 units squared.

So the area under curve is the sum of A1, A2, A3 and A4 which is equal to

9 +11 + 10 + 6 = 36 units squared.

**Deriving Trapezium Rule**

Consider the following figure and answer the following questions

1. How many strips are formed under the curve? There are 5 strips.
2. What are the ordinates of b1 and b2 of the 1st strip and 2nd strip? For the 1st strip, b1 is y0 and b2 is y1 for 2nd strip b1 is y1 and b2 is y2.
3. Calculate the area of each strip/trapezium.
4. Estimate the area under the curve and simplify your answer.
In the previous question we have calculated the area of each strip, now try to estimate the area under the curve the same way.

Area under the curve is defined by the sum of the areas of all strips that is A1+A2+A3+A4+A5. Now replace the expression of each area and add them all together, combine like terms and factorize

So Area = half h into (y0+y1+y1+y2+y2+y3+y3+y4+y4+y5)

As you can see y1 is repeated twice as well as y2, y3 and y4.

Now factorizing further we will get Area = half times h brackets y0+y5+2 into y1+y2+y3+y4 close parentheses and then close brackets

In general if you have n trapezia under the curve and we need to find the area under this curve, the area will be equal to A1+A2+A3+A4+A5+ … +An

So in general area under the curve using trapezium rule is equal to half times h brackets y0+yn +2 into y1 + y2 +… + y n-1 close parentheses then close brackets where y0 is the ordinate of b1 of 1st strip, yn is the ordinate of b2 of last strip and yn-1 is the ordinate of b1 of last strip

**Example 1**

Estimate the area bounded by the curve$ y=\frac{1}{2}x^{2}+5 $, the x-axis, the two lines x=0 and x=5 using trapezium rule, with h=1.

The 1st step is to complete the table below (evaluate y for each x), where y0 is the value of y when x=0 and y1 is the value of y when x=1. Apply trapezium rule to estimate the area under curve

Area = half times h brackets y0+y5 +2 into y1+y2+y3+y4 close parentheses, close brackets

Replace the values h by 1 y0 by 5, y1 by 5.5 and so on

Then the area under curve is **46.25 units square**d.

**Example 2**

Estimate the area under the curve y= 1 over 1+x squared bounded by the x-axis, y axis and the lines x=0 and x=1 with h=0.2.

1st construct a table of x values ranging from 0 to 1 with an interval width h=0.2

In this example, we will use a non-programmable Calculator CASIO fx-82EX to construct table of values in order to reduce time while evaluating.

Steps to construct a table of values using CASIO fx-82EX are as follows press Menu then input 3. The algebraic expression of y or f(x) is 1 over 1 +x squared. In the Calculator press the fraction key then press 1. To move from numerator to denominator press the down arrow then press 1 + in order to type letter x as you can see on the Calculator letter x is in red. In order to activate the red color functions press alpha then parentheses so that the letter x will be activated and printed on the screen. To raise x to power two press x power key followed by 2.

Now, the expression is printed on the Calculator, press equal sign twice i.e. = = so that the Calculator can take the order.

A new screen will appear with title ‘Table Range’, Start means the 1st x value and End means the last x value in the table and Step is for h the interval width.

In this case Start is 0 press = to move to End, press 1 followed by = then input 0.2 in Step. When you finish entering the data press equal sign key twice, i.e. = =. A table will appear on the screen with all y values. Now fill up the table on your paper and apply trapezium rule.

Using CASIO fx-82EX press Menu followed by 1 and input the numerical expression as it is written on your paper

Press fraction key and press 1, then press down arrow to move to denominator and press 2. Press right arrow to move out of the denominator and press times key then input 0.2. Press times key then parentheses and input the values of y0 till y5 accordingly.

Area = half times 0.2 brackets 1 + 0.5 +2 into 0.9615 +0.8621+0.7353+0.6098 close parentheses & close brackets and then press = . The answer is **0.78374 units squared**.

Please take the following assignment to practice what we have explained in this session.

**Question 1**: - Identify real life examples where trapezium rule is applicable

**Question 2**: - A) Complete the table below for the function y equals x squared - 3x - 4

B) Use the table and trapezium rule with 11 strips to estimate the area bounded by the curve x squared - 3x - 4, x = - 4, x = 1 and the x axis.

**Thank you very much!**